6 **a** 100 g

b 225 g

c 400 mL

7 **a** **i** 3 hours

ii 12 hours

iii 4 hours

b 60 km/h

8

Number of students	5	2	3	4	6	7	8
Cost per student ($)	240	600	400	300	200	171.43	150

9 1 hour 20 minutes

10 4 days

11 Zara is incorrect. It will take the same amount of time as 20 minutes is the time the journey takes. It doesn't matter how many people are on the train.

12 $A = 14$, $B = 15$, $C = 49$, $D = 7.5$

13 **a** 169 cm **b** 165 cm

14 **a** 2 houses

b 12 people

c 15 days

d

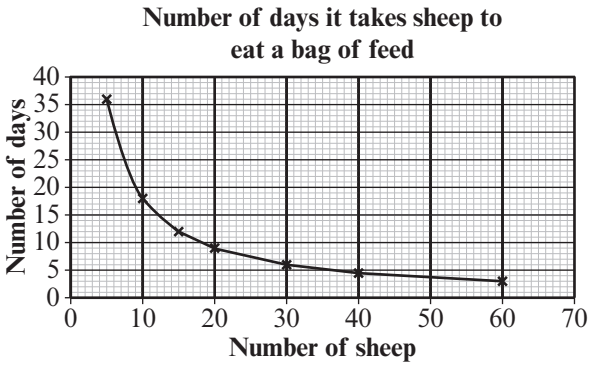
People	Days	Houses
6	20	4
1	120	4
1	30	1
6	60	12
4	60	8

15 **a**

Number of sheep (x)	5	10	15	20	30	40	60
Number of days (y)	36	18	12	9	6	4.5	3
$x \times y$	180	180	180	180	180	180	180

b $x \times y = 180$, yes

c **i** and **iii**



ii No, the points do not form a straight line.

iv Answer between 25 and 26 days (accurate answer is 25.7 to 1 d.p.)

Exercise 12.1

1 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$

c $\frac{1}{4}$ **d** $\frac{5}{12}$

e $\frac{7}{12}$

2 **a** 0.35 **b** 0.2

3 **a** 85% **b** 35% **c** 20%

4 **a** 0.34 **b** 0.7 **c** 0.52

5 **a** **i** 0.45 **ii** 0.65

b 0.85

6 $\frac{5}{12}$

7 **a** 0.4 **b** 0.9

8 **a** 0.01 **b** 0.98

c 0.5 **d** 0.95

e 0.89 **f** 0.81

9 **a** T is 3, 6, 9 or 12 and F is 5 or 10 and these have no common element.

b **i** $\frac{19}{36}$ **ii** $\frac{2}{3}$ **iii** $\frac{17}{36}$

10 **a** **i** $\frac{13}{25}$ **ii** $\frac{12}{25}$ **iii** $\frac{22}{25}$

b 15 is a multiple of both 3 and 5, so the events are not mutually exclusive. Adding the probabilities will not give the correct answer.

Exercise 12.2

1 **a** Learner's own explanation. For example: The score on one dice does not affect the score on the other dice.

b Yes. For example: The score on one dice does not affect the score on the other dice.

c Learner's own explanation. For example: If you get an even number you cannot get an odd number on the same dice and vice versa.

2 **a** Learner's own explanation. For example: The thunderstorm increases the probability that Zara will be late.

b Learner's own explanation. For example: They are not independent. If Arun is late on Monday he will probably make more effort not to be late on Tuesday.

3 **a** **i** $\frac{1}{8}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

b $\frac{1}{4}$ **c** $\frac{1}{4}$

d Learner's own explanation. For example: Whether or not R happens, $P(F) = \frac{1}{4}$.

4 **a** **i** $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$

b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$

c Learner's own explanation. For example: $P(\text{multiple of } 3) = \frac{1}{3}$ both if the number is even and if it is not.

5 **C** is correct. Learner's own explanation. For example: The results of the first three rolls and of the fourth roll are independent.

6 **a** Learner's own explanation. For example: If A happens then $P(B) = \frac{2}{5}$; if A does not happen then $P(B) = \frac{3}{5}$. These are not the same so the events are not independent.

b Yes. Learner's own explanation. For example: This time $P(B) = \frac{1}{2}$ if A happens and if A does not happen.

7 **a** 0.5 **b** 0.5

c Learner's own explanation. For example: The probability of the letter being in the word HEAD is the same, whether it is in the word FACE or not.

d Learner's own explanation. For example: If the letter is in the word FACE, then P(in EACH) = 0.75. If the letter is not in the word FACE it is B, D, G or H and P(in EACH) = 0.25. The probabilities are different, so the events are not independent.

8 **a** Learner's own explanation. For example: If the blue dice is 6 then P(Y) = P(yellow is 6) = $\frac{1}{6}$; If the blue dice is not 6 it will be another number and then P(Y) is again $\frac{1}{6}$. The same probability implies independent events.

b Learner's own explanation. For example: If blue is 6 then P(Z) = P(total of 12) = P(yellow is 6) = $\frac{1}{6}$. If blue is not 6 a total of 12 is impossible and P(Z) = 0. The probabilities are different, so the events are not independent.

9 **a** **i** $\frac{5}{11}$ **ii** $\frac{4}{11}$ **iii** $\frac{2}{11}$

b For all 3 to be the same colour the third must be red.

The probability of this is $\frac{4}{10} = \frac{2}{5}$.

Exercise 12.3

1 **a** 0.32 **b** 0.6

c 0.2 **d** 0.12

2 **a** $\frac{1}{6}$ **b** $\frac{1}{36}$

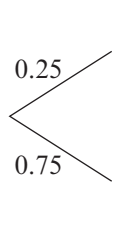
c $\frac{5}{6}$ **d** $\frac{25}{36}$

3 **a** 0.42 **b** 0.3 **c** 0.18

4 **a** $\frac{1}{64}$ **b** $\frac{1}{32}$ **c** $\frac{1}{32}$

d $\frac{5}{32}$ **e** $\frac{25}{64}$

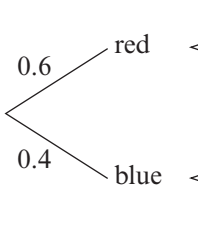
5 a



First	Second	Outcome	
late	late	late, late	$0.25 \times 0.4 = 0.1$
	not late	late, not late	$0.25 \times 0.6 = 0.15$
not late	late	not late, late	$0.75 \times 0.4 = 0.3$
	not late	not late, not late	$0.75 \times 0.6 = 0.45$

b i 0.1 **ii** 0.45 **iii** 0.9

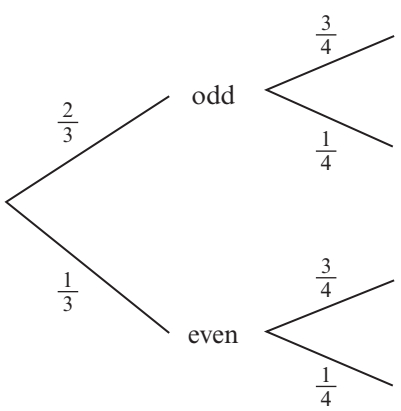
6 a



First	Second	Outcome	
red	red	red, red	$0.6 \times 0.35 = 0.21$
	blue	red, blue	$0.6 \times 0.65 = 0.39$
blue	red	blue, red	$0.4 \times 0.35 = 0.14$
	blue	blue, blue	$0.4 \times 0.65 = 0.26$

b i 0.21 **ii** 0.26 **iii** 0.14

7 a

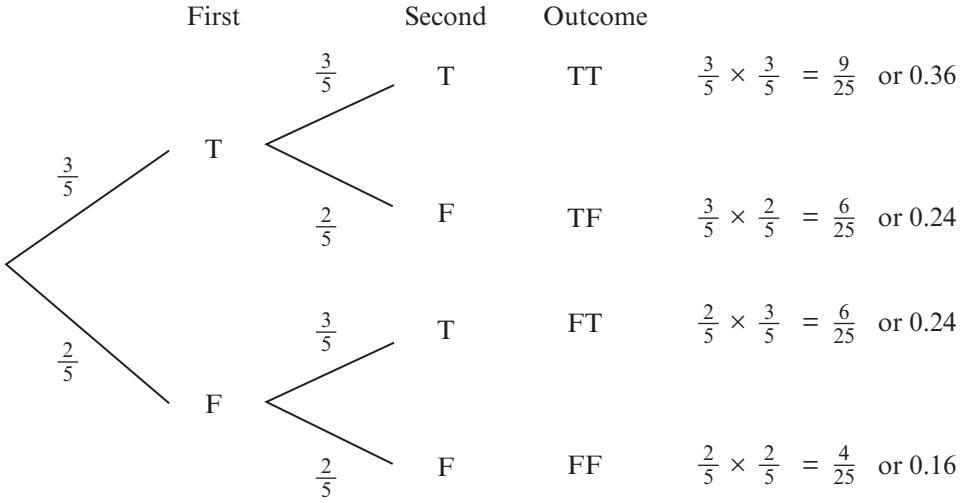


First	Second	Outcome	
odd	odd	odd, odd	$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$
	even	odd, even	$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$
even	odd	even, odd	$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$
	even	even, even	$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

b i $\frac{1}{2}$ **ii** $\frac{1}{12}$

iii $\frac{11}{12}$ **iv** $\frac{5}{12}$

8 **a** Learner's own tree diagram. For example:



The branches for F and T could be reversed.

b **i** $\frac{9}{25}$ or 0.36 **ii** $\frac{21}{25}$ or 0.84

9 0.23. Learner's own method. For example: This could be done with a tree diagram.
$0.8 \times 0.05 + 0.2 \times 0.95 = 0.23$

Exercise 12.4

1 **a** 0.46 **b** 0.18 **c** 0.36

2 **a** 0.45 **b** 0.325

c Yes. Learner's own explanation. For example: 0.325 is quite close to 0.35, so there is no reason to reject the conjecture.

3 **a** 0.58 **b** 0.14

4 **a** $\frac{17}{25} = 0.68$

b **i** 0.7 **ii** 0.785 **iii** 0.782

c 8 is the best estimate. For example: The relative frequency is tending to 0.8 (1 d.p.) and $0.8 \times 10 = 8$

5 **a**

Total	10	20	30	40	50	60	70	80
Silver cars	2	7	11	16	19	23	27	31
Relative frequency	0.2	0.35	0.367	0.4	0.38	0.383	0.386	0.388

b Learner's own graph. Check that the points from the table in part **a** have been plotted correctly.

c Learner's own estimate. 0.38 or 0.39 or 0.4 would be a sensible estimate from the data.

6 **a**

Flips	20	40	60	80	100
Frequency of 2 heads	5	9	11	17	19
Relative frequency	0.25	0.225	0.183	0.2125	0.19

b Learner's own graph. Check that the points from the table in part **a** have been plotted correctly.

c

Flips	20	40	60	80	100
Frequency of 2 heads	4	11	16	20	24
Relative frequency	0.2	0.275	0.267	0.25	0.24

d Learner's own graph. Check that the points from the table in part **c** have been plotted correctly.

7 **a**, **b** and **c** Learner's own results.

d Learner's own graphs.

e Learner's own comparison of relative frequencies with 0.5.

8 **a** and **b** Learner's own results.

c Learner's own comparison of relative frequencies with 0.1666 … (that is $\frac{1}{6}$).

Exercise 13.1

1 **a** $4.5 \times 8 = 36$ km

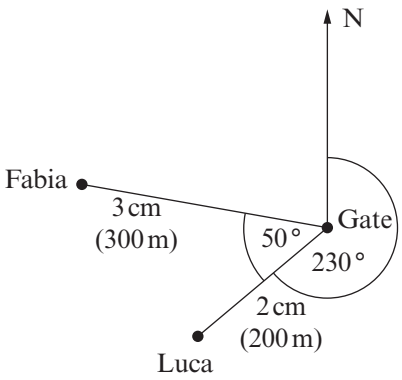
b $18 \div 8 = 2.25$ cm

2 **a** 6 cm. Learner's own diagram. Check that the length of the line is 6 cm and that the angle between the N arrow and the line is 120°.

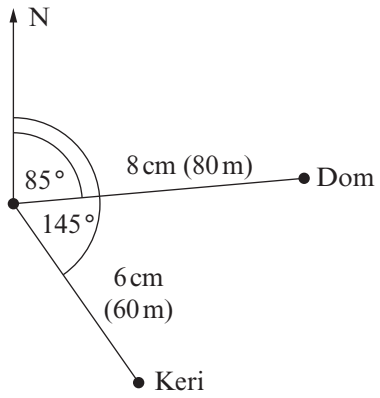
b 8.5 cm. Learner's own diagram. Check that the length of the line is 6 cm and that the angle between the N arrow and the line is 35°.

3 Learner's own diagram. Check that the length of the line is 4 cm and that the angle between the N arrow and the line is 95°.

4



5 **a**

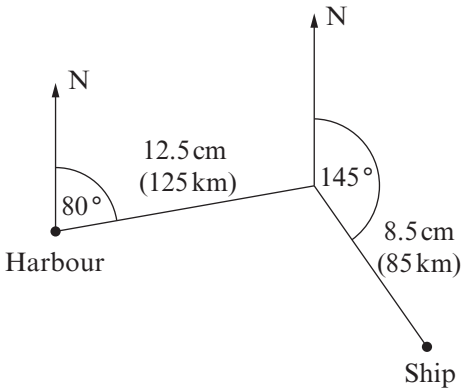


b Learner's own measurement. In the range 70–75 m.

6 No. Learner's own diagram and explanation. For example: The diagram shows that the jeeps are moving in different directions and they will not meet at all.

7 **a**, **b** Yes. Learner's own diagram and explanation. For example: The diagram shows that the ship is much closer to C than B.

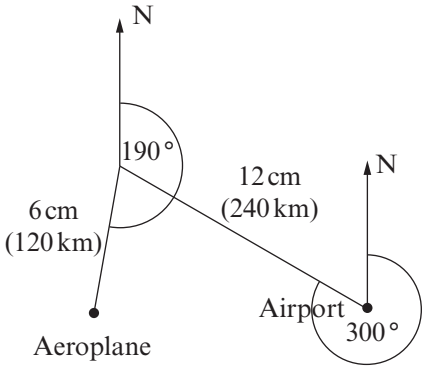
8 **a**



b Learner's own measurement and conversion. In the range 175–180 km.

c Learner's own measurement. In the range 283°–289°.

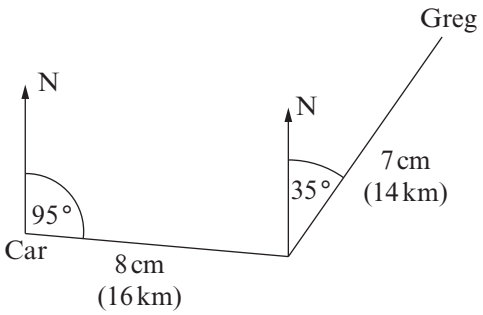
9 **a**



b Learner's own measurement and conversion. In the range 225–230 km.

c Learner's own measurement. In the range 088°–093°.

10 a



b Learner's own measurement and conversion. In the range 25.8–26.2 km.

c Learner's own measurement. In the range 245°–250°.

11 In the range 7.3–7.5 km and 215°–220°.

12 a In the range 45–48 km.

b In the range 52–55 km.

13 a 160°. Learner's own explanation. For example: Triangle is equilateral so angle ABC = 60°. Line BD is parallel to the north arrow so angle ABD = 40° and so angle DBC = 60 − 40 = 20°. Bearing of C from B = 180 − 20 = 160°.

b 280°. Learner's own explanation. For example: Triangle is equilateral so angle ACB = 60°. Line CE is parallel to line BD so angle ECB = angle DBC = 20°. Bearing of C from B = 360 − 20 − 60 = 280°.

c Learner's own accurate sketch and checks.

14 In the range 72–78 km and 338°–342°.

15 a, **b** Learner's own answers. Allow ± 2° on the bearings and ± 2 mm on the distances on the map. For example:

From	To	Bearing	Distance on map (cm)	Distance in real life (m)
Start	A	080°	3.5	700
A	B	100°	4.6	920
B	E	140°	2.4	480
E	F	227°	4.5	900
F	C	345°	3.5	700
C	D	255°	3.6	720
D	Finish	328°	3.3	660

Exercise 13.2

1 a $\left(\frac{1}{3}\times 3,\ \frac{1}{3}\times 6\right) = (3\div 3,\ 6\div 3) = (1, 2)$

b $\left(\frac{2}{3}\times 3,\ \frac{2}{3}\times 6\right) = (3\div 3\times 2,\ 6\div 3\times 2) = (2, 4)$

2 a $\left(\frac{1}{4}\times 4,\ \frac{1}{4}\times 12\right) = (4\div 4,\ 12\div 4) = (1, 3)$

b $\left(\frac{3}{4}\times 4,\ \frac{3}{4}\times 12\right) = (4\div 4\times 3,\ 12\div 4\times 3) = (3, 9)$

3 **H** and **iii**, **I** and **ii**, **J** and **vi**, **K** and **i**, **L** and **v**, **M** and **iv**

4 **a** (9, 12)

b

Letter	A	B	C	D	E	F	K	R	W
Position in alphabet	1st	2nd	3rd	4th	5th	6th	11th	18th	23rd
x-coordinate	$1 \times 3 = 3$	$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$	$6 \times 3 = 18$	$11 \times 3 = 33$	$18 \times 3 = 54$	$23 \times 3 = 69$
y-coordinate	$1 \times 4 = 4$	$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$	$6 \times 4 = 24$	$11 \times 4 = 44$	$18 \times 4 = 72$	$23 \times 4 = 92$
Coordinate pair	(3, 4)	(6, 8)	(9, 12)	(12, 16)	(15, 20)	(18, 24)	(33, 44)	(54, 72)	(69, 92)

c **i** The x-coordinates are the numbers in the 3 times table. To work out the x-coordinate of any letter, multiply the position number of the letter in the alphabet by 3.

ii The y-coordinates are the numbers in the 4 times table. To work out the y-coordinate of any letter, multiply the position number of the letter in the alphabet by 4.

5 **a** **B** (4, 5) **b** **C** (12, 15)

c **A** (3, 2) **d** **C** (15, 10)

6 **a** (10, 16) **b** (15, 24)

c Learner's own explanation. For example: E is the 5th letter of the alphabet, so has coordinates $(5 \times 5, 5 \times 8) = (25, 40)$.

d T is the 20th letter of the alphabet, so has coordinates $(20 \times 5, 20 \times 8) = (100, 160)$.

e nth letter of the alphabet has coordinates $(5n, 8n)$.

7 H (28, 36)

8 Difference in x-coordinates $= 10 - 1 = 9$ $\quad \frac{1}{3} \times 9 = 3$

Difference in y-coordinates $= 13 - 1 = 12$ $\quad \frac{1}{3} \times 12 = 4$

E $=$ C$(1, 1) + (3, 4) = (1 + 3, 1 + 4) = (4, 5)$

9 **a** Difference in x-coordinates $= 7 - 2 = 5$ $\quad \frac{2}{5} \times 5 = 2$

Difference in y-coordinates $= 18 - 3 = 15$ $\quad \frac{2}{5} \times 15 = 6$

H $=$ F$(2, 3) + (2, 6) = (2 + 2, 3 + 6) = (4, 9)$

b Learner's own check by drawing a diagram.

10 **a** Yes. Learner's own justification. For example:

x-coordinates: $\frac{2}{3}$ of the way $= 8$, so $\frac{1}{3}$ of the way $= 8 \div 2 = 4$, so $\frac{3}{3}$ of the way $= 4 \times 3 = 12$.

y-coordinates: $\frac{2}{3}$ of the way $= 10$, so $\frac{1}{3}$ of the way $= 10 \div 2 = 5$, so $\frac{3}{3}$ of the way $= 5 \times 3 = 15$.

So, B is the point (12, 15).

b No. Learner's own justification. For example: OA is $\frac{2}{3}$ of OB so ratio OA : AB $= \frac{2}{3} : \frac{1}{3} = 2 : 1$.

11 a Learner's own diagram. Coordinate grid with points A (2, 7), B (10, 3), C (5, 3) and D (11, 5). Line segments AB and CD drawn. Point where AB crosses CD labelled 'E'.

b E (8, 4)

c Midpoint of CD =
$\left(\frac{5+11}{2}, \frac{3+5}{2}\right) = \left(\frac{16}{2}, \frac{8}{2}\right) = (8, 4) = \text{E}$

d $\frac{3}{4}$ of the way along AB:
Difference in x-coordinates $= 10 - 2 = 8$
$\frac{3}{4} \times 8 = 6$

Difference in y-coordinates $= 3 - 7 = -4$
$\frac{3}{4} \times -4 = -3$

So, A$(2, 7) + (6, -3) = (2 + 6, 7 - 3)$
$= (8, 4) = \text{E}$.

12 Difference in x-coordinates =
$-4 - -9 = -4 + 9 = 5$
$\frac{3}{5} \times 5 = 3$
Difference in y-coordinates =
$-2 - -12 = -2 + 12 = 10$
$\frac{3}{5} \times 10 = 6$

So, K$(-9, -12) + (3, 6) = (-9 + 3,$
$-12 + 6) = (-6, -6)$.

13 a S $(-1, -1)$

b T $(0, 1)$

Learner's own working. For example:

Difference in x-coordinates is $2 - -4 = 6$.

Difference in y-coordinates is $5 - -7 = 12$.

There are 6 points after P, so the x-coordinates increase by $6 \div 6 = 1$ for each point, and the y-coordinates increase by $12 \div 6 = 2$ for each point.

Points:	P	Q	R	S	T	U	V
x-coordinates	−4	−3	−2	−1	0	1	2
y-coordinates	−7	−5	−3	−1	1	3	5

Exercise 13.3

1 a, b Learner's own diagram. Coordinate grid. Triangle A with vertices (1, 4), (2, 3) and (2, 5). Triangle B with vertices (4, 3), (5, 4) and (4, 5). Triangle C with vertices (6, 2), (7, 3) and (6, 4).

2 a, b Learner's own diagram. Coordinate grid. Rectangle A with vertices (1, 4), (3, 4), (3, 5) and (1, 5). Rectangle B with vertices (3, 4), (4, 4), (4, 6) and (3, 6). Rectangle C with vertices (3, 0), (4, 0), (4, 2) and (3, 2).

3 **a** and **ii**, **b** and **iii**, **c** and **i**

4 a A to B is a reflection in the mirror line $x = 3$.

b A to C is a translation $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

c A to D is a rotation 90°, anticlockwise, centre (1, 4).

d B to E is a reflection in the mirror line $y = 5$.

e E to C is a rotation 180°, centre (6, 6).

f C to F is a translation $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$.

5 a–c Learner's own diagram. Coordinate grid. Triangle A with vertices (6, 4), (7, 6) and (5, 6). Triangle B with vertices (1, 1), (3, 1) and (2, 3). Triangle C with vertices (3, 1), (5, 2) and (3, 3). Triangle D with vertices (5, 0), (7, 0) and (6, 2).

6 a–d Learner's own diagram. Coordinate grid. Triangle A with vertices (−1, −4), (−3, −2) and (−4, −4). Triangle **a** with vertices (3, −3), (6, −3) and (5, −1). Triangle **b** with vertices (0, 0), (−3, 0) and (−1, 2). Triangle **c** with vertices (0, 2), (−2, 3) and (0, 5). Triangle **d** with vertices (3, 3), (6, 3) and (5, 5).

7 She has only got part **b** correct. Learner's own explanations. For example: The object and its image are always congruent, so answers are:

a Corresponding lengths are the same.

b Corresponding angles are the same.

c The object and the image are congruent.

8 a Translation $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

b Translation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

c Translation $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$

d Reflection in line $y = 1$.

e Reflection in line $x = 0$ (or y-axis).

f Reflection in line $y = -2$.

g Reflection in line $x = -2$.

h Rotation 90° clockwise, centre (0, −4).

i Rotation 90° anticlockwise, centre (0, 0).

j Rotation 180°, centre (0, 2).

9 a i Rotation 90° anticlockwise about (−1, 3).

ii Translation $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

iii Reflection in the line $x = -1$.

iv Reflection in the line $x = -3.5$.

b i Check learners' own combinations of at least two transformations. For example:

Rotation 90° anticlockwise about (−2, 3) followed by translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

ii Check learners' combinations of at least two transformations. For example:

Reflection in line $x = 3$, followed by translation $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

10 Zara is not correct. Arun is correct. Learner's own diagrams.

11 a A to C **b** A to B

c A to D **d** B to E

12 a Learner's own diagram. Check that each of the transformed shapes are drawn correctly. The final image should have vertices (4, 0), (5, 0), (5, 1), (5, −1) and (4, 1).

b A translation $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$

Exercise 13.4

1 a Learner's own diagram. Enlarged triangle with vertices (3, 1), (5, 1) and (3, 5).

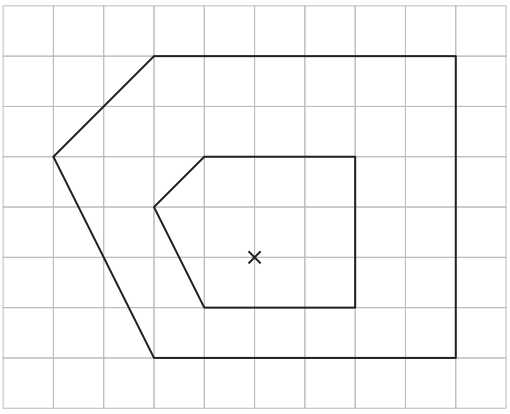
b Learner's own diagram. Enlarged triangle with vertices (1, 0), (5, 2) and (1, 2).

2 a Learner's own diagram. Enlarged square with vertices (0, 1), (3, 1), (3, 4) and (0, 4).

b Learner's own diagram. Enlarged triangle with vertices (0, 2), (3, 2) and (0, 5).

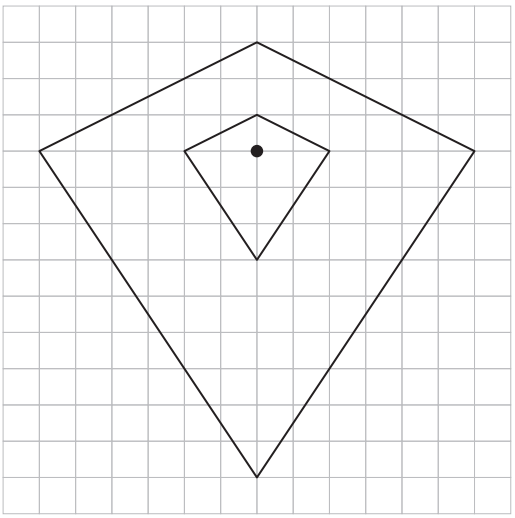
3 Learner's own diagram. The enlarged right-angled triangle should have the corresponding vertex on the cross, base length eight squares and height four squares.

4

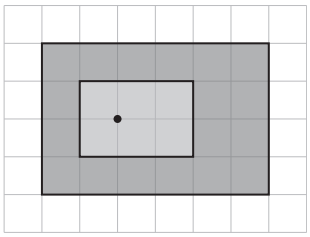


5 a Learner's own explanation. For example: The top vertex of the kite is only two squares from the centre of enlargement, not three. All the other vertices are in the correct position.

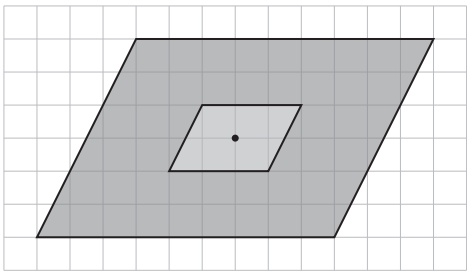
b



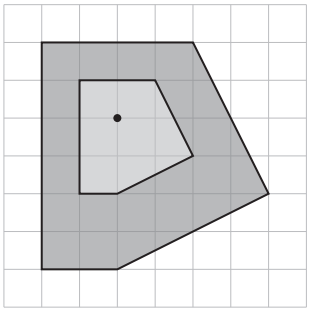
6 a scale factor 2



b scale factor 3



c scale factor 2



7 **a** Learner's own diagram. Shape B with vertices (6, 0), (10, 0), (10, 2), (8, 4) and (6, 4).

b Learner's own diagram. Shape C with vertices (0, 1), (6, 1), (6, 4), (3, 7) and (0, 7).

c Learner's own diagram. Shape D with vertices (1, 0), (9, 0), (9, 4), (5, 8) and (1, 8).

8 **a** Learner's own diagram. Enlarged shape with vertices as given in part **b**.

b (1, 1), (1, 5), (7, 5), (3, 1)

9 Perimeter of N = $12 \times 4 = 48$ cm

Area of N = $9 \times 4^2 = 144$ cm^2

10 Perimeter of Z = 36 cm

Area of Z = 72 cm^2

11 **a** Enlargement scale factor 3, centre (6, 2).

b Enlargement scale factor 2, centre (3, 5).

12 Enlargement scale factor 3, centre (6, 1).

13 Enlargement scale factor 2, centre (3, 4).

14 **a** Enlargement scale factor 2, centre (2, 4).

b $C = \pi d = 4\pi = 12.566$ cm (3 d.p.)

c $C = 2 \times 12.56637\ldots = 25.133$ cm (3 d.p.)

d $C = \pi d = 8\pi = 25.133$ cm (3 d.p.)

e $A = \pi r^2 = \pi \times 2^2 = 4\pi = 12.566$ cm^2 (3 d.p.)

f $A = 2^2 \times 12.566 = 50.265$ cm^2 (3 d.p.)

g $A = \pi r^2 = \pi \times 4^2 = 16\pi = 50.265$ cm^2 (3 d.p.)

15 **a** Arun is incorrect. He has multiplied the perimeter of H by 3 instead of dividing by 3.

Perimeter = $36 \div 3 = 12$ cm

b Area = $54 \div 3^2 = 6$ cm^2

Exercise 14.1

1 **a** Volume = area of cross-section × length
$= 20 \times 8 = 160$ cm^3

b Volume = area of cross-section × length
$= 15 \times 6 = 90$ cm^3

c Volume = area of cross-section × length
$= 12 \times 9 = 108$ cm^3

d Volume = area of cross-section × length
$= 30 \times 12 = 360$ cm^3

2 **a** Area of cross-section = area of rectangle
= base × height $= 8 \times 4 = 32$ cm^2
Volume = area of cross-section × length
$= 32 \times 10 = 320$ cm^3

b Area of cross-section = area of triangle
$= \frac{1}{2} \times$ base × height $= \frac{1}{2} \times 6 \times 5 = 15$ cm^2
Volume = area of cross-section × length
$= 15 \times 7 = 105$ cm^3

c Area of cross-section = area of circle
$= \pi \times r^2 = \pi \times 4^2$
$= \pi \times 16 = 50.265\ldots$ cm^2
Volume = area of cross-section × length
$= 50.265\ldots \times 11 = 552.92$ cm^3

3 **a** 150 cm^3

b 129.6 cm^3

c 427.5 cm^3

4

	Area of cross-section	Length of prism	Volume of prism
a	8.4 cm^2	20 cm	168 cm^3
b	24 cm^2	6.5 cm	156 cm^3
c	58 m^2	5.7 m	330.6 m^3
d	56.85 mm^2	62 mm	3524.7 mm^3

5 **a** $480\,cm^3$ **b** $480\,cm^3$

c $675\,cm^3$

6 Learner's own answer and explanation. For example: Timo has used the diameter instead of the radius in the volume formula. Correct answer is $226\,cm^3$ (3 s.f.)

7 **a** $754.0\,cm^3$ **b** $492.6\,cm^3$

c $42\,411.5\,mm^3$

8

	Radius of circle	Area of circle	Height of cylinder	Volume of cylinder
a	7 cm	153.94 cm^2	12 cm	1847.26 cm^3
b	1.5 m	7.07 m^2	2.4 m	16.96 m^3
c	9 cm	254.47 cm^2	7.51 cm	1910 cm^3
d	2.19 m	15 m^2	3.8 m	57 m^3
e	4.55 mm	65 mm^2	22 mm	1430 mm^3

9 **a** $x = 4.3$

b $x = 3.3$

c $x = 2.1$

10 **a** $729\,cm^3$

b 13 851 g

c \$96 957

11 The smallest. Learner's own explanation. For example: In the smallest tin you get $233\,cm^3$ per dollar, compared with $201\,cm^3$ per dollar for the medium tin and $225\,cm^3$ per dollar for the large tin.

Exercise 14.2

1 **a** Square base: $10 \times 10 = 100$

Triangular faces: $4 \times \left(\frac{1}{2} \times 10 \times 8\right) = 160$

Total $= 100 + 160 = 260\,cm^2$

b Triangular face: $\frac{1}{2} \times 20 \times 17 = 170$

Total $= 4 \times 170 = 680\,cm^2$

2 **a** Front and back: $2 \times (4 \times 12) = 96$

Sides: $2 \times (4 \times 10) = 80$

Top and base: $2 \times (12 \times 10) = 240$

Total $= 96 + 80 + 240 = 416\,cm^2$

b Triangle ends: $2 \times \left(\frac{1}{2} \times 24 \times 5\right) = 120$

Sides: $2 \times (13 \times 8) = 208$

Base: $24 \times 8 = 192$

Total $= 120 + 208 + 192 = 520\,cm^2$

c Triangle ends: $2 \times \left(\frac{1}{2} \times 12 \times 9\right) = 108$

Sloping face: $15 \times 12.5 = 187.5$

Back face: $9 \times 12.5 = 112.5$

Base: $12 \times 12.5 = 150$

Total $= 108 + 187.5 + 112.5 + 150 = 558\,cm^2$

3 Area of circle $= \pi r^2$

$= \pi \times 3^2$

$= 28.27\,cm^2$ (2 d.p.)

Circumference of circle $= \pi d$

$= \pi \times 6$

$= 18.85\,cm$ (2 d.p.)

Area of rectangle = circumference of circle × 8

$= 18.85 \times 8$

$= 150.80\,cm^2$ (2 d.p.)

Total area = 2 × area of circle + area of rectangle

$= 2 \times 28.27 + 150.80$

$= 207\,cm^2$ (3 s.f.)

4 **a** $477.5\,cm^2$

b $401.1\,cm^2$

c $8482.3\,mm^2$

5 The tetrahedron has the greater surface area.

Tetrahedron: SA $= 448\,cm^2$,
Cylinder: SA $= 427.25\ldots\,cm^2$

$448\,cm^2 > 427.25\ldots\,cm^2$

6 $151\,cm^2$ (3 s.f.)

7 **a** $3840\,mm^2$

b $1548\,cm^2$

8 $344\,cm^2$ (3 s.f.)

9 Carlos is incorrect. Learner's own working and explanations. For example:

The surface area of the polytunnel =
$\frac{1}{2}\times\left(2\times\left(\pi\times4.5^2\right)+\pi\times9\times27\right)=445.32\ldots\ \text{m}^2$

Carlos will need more than 445 m² of plastic to make the polytunnel as the total surface area is more than 445 m² and he will need to allow extra plastic for overlaps at the edges, wastage, etc.

10 **a** Learner's own answers.

b Prism B has a greater surface area than prism A.

c Prism A:

$\text{SA}=2\times\frac{1}{2}\times\pi\times5^2+\frac{1}{2}\times\pi\times10\times15+10\times15$
$=464\,\text{cm}^2\ (3\text{ s.f.})$

Prism B:

$\text{SA}=2\times\frac{1}{4}\times\pi\times8^2+\frac{1}{4}\times\pi\times16\times13+2\times8\times13$
$=472\,\text{cm}^2\ (3\text{ s.f.})$

$\text{Difference}=472-464=8\,\text{cm}^2$

Exercise 14.3

1 **a** and **d**

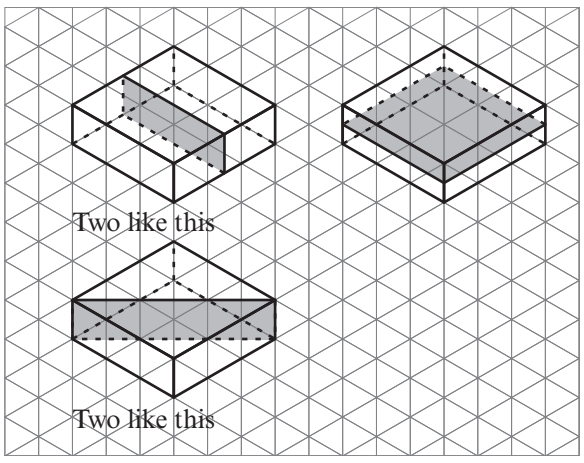
2 **a** **A** and **F** **b** **B** and **D**

c **C**

3 **a** Learner's own diagram. One vertical plane of symmetry, splitting the shape into two congruent shapes.

b Learner's own diagram. One vertical plane of symmetry, splitting the shape into two congruent shapes.

4



5 **a** There are two vertical and one horizontal planes of symmetry.

b Learner's own diagram. Two vertical and one horizontal planes of symmetry, each plane splitting the shape into two congruent shapes.

6 **a**

2D regular polygon	Number of lines of symmetry	3D pyramid	Number of planes of symmetry
triangle	3	triangular	3
square	4	square	4
pentagon	5	pentagonal	5
hexagon	6	hexagonal	6
octagon	8	octagonal	8

b The number of lines of symmetry of a regular 2D polygon is the same as the number of planes of symmetry of its matching 3D pyramid. Learner's own explanation. For example: This is because all the lines of symmetry become vertical planes of symmetry, but the triangular sides of the pyramid meet at a point so there are no horizontal lines of symmetry.

c **i** 10 **ii** 12

7 **a** Learner's own diagram. Check that the drawn part is a reflection of the shape drawn in the question.

b One

c Learner's own diagram. One plane of symmetry splitting the shape into two copies of the shape in the question. The other plane is perpendicular to the first, splitting the shape into two congruent shapes.

8 **a** Learner's own diagram. Check that the drawn part is a reflection of the shape drawn in the question.

b Cube

c 9

Exercise 15.1

1 a

Time, t (seconds)	Frequency	Midpoint
$0 < t \leqslant 10$	1	5
$10 < t \leqslant 20$	6	15
$20 < t \leqslant 30$	8	25
$30 < t \leqslant 40$	11	35
$40 < t \leqslant 50$	14	45
$50 < t \leqslant 60$	5	55

b Learner's own frequency polygon. Same axes and labels as in the diagram in the question. Points (5, 1), (15, 6), (25, 8), (35, 11), (45, 14) and (55, 5) plotted and joined with straight lines.

2 a

Height, h (cm)	Frequency	Midpoint
$260 \leqslant h < 280$	3	270
$280 \leqslant h < 300$	7	290
$300 \leqslant h < 320$	9	310
$320 \leqslant h < 340$	1	330

b Learner's own frequency polygon. Same axes and labels as in the diagram in the question. Points (270, 3), (290, 7), (310, 9) and (330, 1) plotted and joined with straight lines.

3 a 32

b

Height, t (cm)	Frequency	Midpoint
$10 \leqslant t < 12$	4	11
$12 \leqslant t < 14$	16	13
$14 \leqslant t < 16$	7	15
$16 \leqslant t < 18$	5	17

c Learner's own frequency polygon. Make sure that they use suitable axes labels and scales. Points (11, 4), (13, 16), (15, 7) and (17, 5) plotted and joined with straight lines.

d $\frac{20}{32} = \frac{5}{8}$

e Learner's own answer and explanation. For example: Zara is incorrect. You don't know from the frequency polygon what the fastest time is. All you can say is that the fastest time is between 10 and 12 minutes.

4 a

Time, t (seconds)	Tally	Frequency	Midpoint
$0 < t \leqslant 10$	II	2	5
$10 < t \leqslant 20$	𝍸	5	15
$20 < t \leqslant 30$	𝍸 III	8	25
$30 < t \leqslant 40$	IIII	4	35
$40 < t \leqslant 50$	I	1	45

b Learner's own frequency polygon. Make sure that they use suitable axes labels and scales. Points (5, 2), (15, 5), (25, 8), (35, 4) and (45, 1) plotted and joined with straight lines.

5 a 50

b

Wednesday		
Height, h (cm)	Frequency	Midpoint
$120 \leqslant h < 140$	4	130
$140 \leqslant h < 160$	6	150
$160 \leqslant h < 180$	22	170
$180 \leqslant h < 200$	18	190

Saturday		
Height, h (cm)	Frequency	Midpoint
$120 \leqslant h < 140$	25	130
$140 \leqslant h < 160$	16	150
$160 \leqslant h < 180$	7	170
$180 \leqslant h < 200$	2	190

c Learner's own diagram showing two frequency polygons on one set of axes. Make sure that they use suitable axes labels and scales. Make sure that each polygon is labelled clearly. Wednesday points (130, 4), (150, 6), (170, 22) and (190, 18) plotted and drawn with straight lines. Saturday points (130, 25), (150, 16), (170, 7) and (190, 2) plotted and joined with straight lines.

d Learner's own answer and explanation. For example: On Saturday there were fewer taller people and more shorter people. There were only two people with a height between 180 cm and 200 cm on Saturday compared with 18 on Wednesday. There were 25 people with a height between 120 cm and 140 cm on Saturday compared with four on Wednesday.

6 a Learner's own diagram showing two frequency polygons on one set of axes. Make sure that they use suitable axes labels and scales. Make sure that each polygon is labelled clearly. Falcons Club points plotted at (2.5, 4), (7.5, 24), (12.5, 18), (17.5, 12) and (22.5, 10) and joined with straight lines. Harriers Club points plotted at (2.5, 10), (7.5, 8), (12.5, 10), (17.5, 26) and (22.5, 16) and joined with straight lines.

b Learner's own comments. For example: The most popular training time for the Falcons Club was between 5 and 10 hours, whereas for the Harriers Club it was between 15 and 20 hours. In the Falcons Club only 22 athletes trained for more than 15 hours a week compared with 42 athletes from the Harriers Club.

c Falcons Club 68, Harriers Club 70.

d Learner's own answer and explanation. For example: Yes, because the number of athletes surveyed at each club was nearly the same.

7 a Learner's own frequency polygon. Make sure that they use suitable axes labels and scales. Points (7.05, 2), (7.15, 12), (7.25, 14), (7.35, 9), (7.45, 7) and (7.55, 6) plotted and joined with straight lines.

b i

Mass, m (kg)	Frequency
$7.0 \leqslant m < 7.2$	14
$7.2 \leqslant m < 7.4$	23
$7.4 \leqslant m < 7.6$	13

ii Learner's own frequency polygon. Make sure that they use suitable axes labels and scales. Points (7.1, 14), (7.3, 23) and (7.5, 13) plotted and joined with straight lines.

c Learner's own answers and explanations. For example: The first frequency polygon gives you better information because there are more groups so it shows you more information on the mass of the girls. The second frequency polygon only has three groups so less information can be taken from the graph.

d i 12

ii No, Sienna cannot fill in the correct frequencies in her table. Learner's own explanation. For example: From the first table Sienna knows that there are two girls with a mass between 7.0 and 7.1 kg. However, this does not tell her how many girls had masses between 7.0 and 7.05 kg and how many girls had masses between 7.05 and 7.1 kg, so it is impossible for her to complete her table. She would have to find the original data, before it was grouped, in order to group it the way she wants to.

8 Learner's own answers. For example:

a

Time to solve maths problem, t (seconds)	Tally	Frequency	Midpoint
$20 \leqslant t < 30$	𝍸 𝍸	10	25
$30 \leqslant t < 40$	𝍸 𝍸 𝍸 𝍸 𝍸	25	35
$40 \leqslant t < 50$	𝍸 𝍸 𝍸 |||	18	45
$50 \leqslant t < 60$	𝍸 ||	7	55

b Learner's own frequency polygon. Make sure that they use suitable axes labels and scales. Points (25, 10), (35, 25), (45, 18) and (55, 7) plotted and joined with straight lines.

c Learner's own comments. For example: Most students took less than 40 seconds to solve the puzzle.

9

Mass, m (kg)	Frequency	Midpoint
$3.6 \leqslant m < 3.8$	8	3.7
$3.8 \leqslant m < 4.0$	12	3.9
$4.0 \leqslant m < 4.2$	9	4.1
$4.2 \leqslant m < 4.4$	11	4.3
$4.4 \leqslant m < 4.6$	5	4.5

Exercise 15.2

1 a Learner's own scatter graph with axes labelled as in the graph in the question. Points (3, 5), (11, 10), (18, 18), (19, 20), (5, 6), (20, 18), (14, 16), (8, 9), (9, 11), (7, 6), (5, 5), (16, 15), (10, 11), (9, 7) and (16, 16) marked with crosses.

b **A**. Learner's own explanation. For example: The scatter graph is showing a positive correlation. This happens when as one value increases, the other value also increases. In this case, as the French results increase, the Spanish results also increase.

2 a Learner's own scatter graph with axes labelled as in the graph in the question. Points (3, 19), (10, 11), (15, 7), (8, 11), (10, 10), (13, 9), (4, 17), (16, 5), (12, 10), (8, 14), (17, 2), (11, 9), (5, 15), (20, 4) and (7, 13) marked with crosses.

b **B**. Learner's own explanation. For example: The scatter graph is showing a negative correlation. This happens when as one value increases, the other value decreases. In this case, as the art results increase, the science results decrease.

3 a Learner's own scatter graph. Horizontal axis labelled 'Hours reading'. Vertical axis labelled 'Spelling test score'. Both axes shown from 0 to 25. Points (4, 6), (13, 12), (20, 20), (9, 8), (18, 17), (1, 2), (11, 13), (8, 10), (18, 19), (2, 3), (15, 16), (10, 12), (4, 5), (14, 12) and (7, 7) plotted.

b Positive correlation. The more hours reading a student does, the better their spelling test score.

c Learner's own line of best fit. Strong positive correlation.

d Learner's own estimate from their line of best fit.

e No. It is not a good idea to use the line of best fit to make predictions outside the range of the data, because you do not know what happens beyond the data you are given.

4 a Learner's own scatter graph. Horizontal axis labelled 'Number of packets of cookies sold' and shown from 0 to 30. Vertical axis labelled 'Number of packets of oranges sold' and shown from 0 to 30. Points (15, 12), (12, 22), (26, 14), (22, 7), (8, 28), (25, 27), (16, 25), (14, 18), (9, 17) and (28, 25) plotted.

b No correlation. The number of packets of cookies sold has no relationship to the number of packets of oranges sold.

5 a Learner's own answers. For example: Negative correlation because learners are often good at maths and science or languages and drama, but are not good at all these subjects.

b Learner's own scatter graph. Horizontal axis labelled 'Maths result' and shown from 0 to 100. Vertical axis labelled 'Drama result' and shown from 0 to 100. Points (72, 27), (34, 62), (81, 19), (57, 41), (32, 66), (78, 25), (65, 37), (67, 32), (53, 59), (61, 48), (35, 63), (42, 59), (55, 40), (79, 35) and (31, 77) plotted.

c Strong negative correlation. The better the students' result in maths, the worse their drama result.

d Learner's own answer.

e Learner's own line of best fit.

f Learner's own estimate from their line of best fit.

g No, because 10% for drama lies outside the range of the data we are given, so we cannot predict what will happen.

6 Learner's own explanation. For example: It is a coincidence that the graph shows a negative correlation. While it might be true that if you have no hair or short hair you need a hat to keep your head warm or protect it from the sun, it does not mean that you are going to buy lots of hats. In this study, the people with longer hair might have big families, and so they bought lots of hats for their family members. The number of hats you need does not depend on the length of your hair. It depends on whether you like to wear hats or not.

7 a Positive correlation. The better the score in algebra, the better the score in geometry.

b 4 c 10 d 13

e Learner's own diagram. Scatter graph from the question with the point (10, 13) plotted.

f Learner's own line of best fit. Make sure the line of best fit passes through the point (10, 13).

g **i** Learner's own estimate using their line of best fit. For example: 8.

ii Learner's own estimate using their line of best fit. For example: 11.

8 **a** Negative correlation. The further the house is from the railway station, the lower its value is.

b The house that doesn't fit the trend is worth \$146 000 and is 6 km from the railway station. Learner's own explanation. For example: The house might not be in a very good state of repair, which is why it isn't worth as much as it should be.

c $y = 6.4$ km

Exercise 15.3

1 **a** **Key:** 0 | 5 means 5

0	5 7 9 9
1	5 4 6 0 8 7 9
2	6 2 5 1 2 7 3 0 4

b **Key:** 0 | 5 means 5

0	5 7 9 9
1	0 4 5 6 7 8 9
2	0 1 2 2 3 4 5 6 7

2 Unordered:

Key: 0 | 6 means 6

0	6 8 4 6
1	8 2 1 4 7 3 9 0 2
2	1 8 1 0 8 1 2

Ordered:

Key: 0 | 4 means 4

0	4 6 6 8
1	0 1 2 2 3 4 7 8 9
2	0 1 1 1 2 8 8

3

Spanish test results for class 9R

	9 9 7 5 | 0
	9 8 7 6 5 4 0 | 1
7 6 5 4 3 2 2 1 0 |	2

Key for class 9R: 5 | 0 means 5

Spanish test results for class 9T

4 6 6 8	
0 1 2 2 3 4 7 8 9	
0 1 1 1 2 8 8	

Key for class 9T: 0 | 4 means 4

4 **a**

June		August
	0 | 2	
9 7	3	6 7 8
6 6 6 5 3 3 2 1 0	4	0 3 7
	5	0 2 4 5 6 8 8
8 0	6	2

Key: For June, 0 | 2 means 20 customers

For August, 3 | 6 means 36 customers

b

	i Mode	ii Median	iii Range	iv Mean
June	46	43	48	44
August	58	51	26	49

c Learner's own answers. For example: In August the mode, median and mean are all greater than in June, showing that on average there are more customers. The range, however, is smaller in August than in June, showing that there is more variation in the numbers of customers riding in June.

d Learner's own answers. For example: Yes, because the mode, median and mean are all greater in August than in June.

5 **a** **i–iv**

	i Mode	ii Median	iii Range	iv Mean
Girls' times	27.3	26.05	2.6	26.1
Boys' times	26.5	27.4	3.6	27.3

b Learner's own answer. For example: The range is larger for the boys, showing that their times are more varied. The girls have a lower median and mean which shows that using these averages they were faster at solving the puzzle.

c **i** The mode, as the boys' mode is lower than the girls', which makes them appear faster.

ii The median or the mean, as the girls' median and mean are lower than the boys', so the girls were faster.

d Learner's own answer. For example: The girls, as their median and mean are lower, therefore they were faster than the boys.

6

	Location A	Location B
a	$\frac{1}{4}$	$\frac{2}{3}$
b	$33\frac{1}{3}\%$	0%
c	Range = 305 g (most variation)	Range = 295 g
d	Mean = 792.5 g, Median = 790 g	Mean = $658\frac{1}{3}$ g, Median = 652.5 g

e Learner's own answers. For example: Location A because the mean and median mass of potatoes was greater than location B. The range was very similar showing that the variation in the mass of potatoes was similar at both locations.

7 a

Top shelf				Middle shelf
	9 4	10		
	9 2 2	11	5	
9 8 7 6 5 4 2 0		12	0 7 9	
	0	13	0 2 6 8 9 9	
		14	2 4 5 7	

Key: For the top shelf, 4 | 10 means 104 boxes of cereal

For the middle shelf, 11 | 5 means 115 boxes of cereal

b

	Mode	Median	Range	Mean
Top shelf	112	123	26	120.5
Middle shelf	139	137	32	134.5

Learner's own answers. For example: The sales of cereal were better on the middle shelf as on average more boxes were sold (the mean, median and mode were all greater on the middle shelf than the top shelf). The sales on the middle shelf were more varied, but included the largest number of boxes sold on one day. The smallest number of boxes sold on one day were on the top shelf.

8 a Missing numbers from the diagram: top row 0, second top row 5, third top row 2 and 8, bottom row 5. Missing numbers from the table: Class 9T row 144 and 145, Class 9R row 149.

b Learner's own comments. For example:

On average, using the median and mean, class 9R were taller than class 9T. Class 9T had more variation in heights, and their modal height was taller than class 9R.

Exercise 15.4

1 a i $12 \leqslant t < 14$ **ii** $12 \leqslant t < 14$

b 8 minutes

c

Midpoint	Frequency	Midpoint × frequency
11	7	11 × 7 = 77
13	12	13 × 12 = 156
15	10	15 × 10 = 150
17	2	17 × 2 = 34
Totals:	**31**	**417**

Estimate of mean = $\frac{417}{31}$ = 13 minutes

d Learner's own explanation. For example: The answers for the range and mean are only estimates because the data is grouped and you do not have the individual values of the data.

2 a i $290 \leqslant h < 310$

ii $290 \leqslant h < 310$

b Learner's own explanation. For example: you can only give the modal class and class where the median lies, because the data is grouped and you do not know the individual values.

c i 80 cm **ii** 290 cm

3 a 150 men, 140 women

b

	Modal class interval	Class interval where the median lies	Estimate of mean
Men	$20 \leqslant a < 30$	$40 \leqslant a < 50$	39.6
Women	$50 \leqslant a < 60$	$40 \leqslant a < 50$	42.5

c Learner's own answer. For example: The mean age of the women is approximately three years more than the men. The median age lies in the same interval for both men and women. The modal class interval for the men is a lot younger than for the women.